KENDRIYA VIDYALAYA 1 TAMBARAM

PRACTICE PAPER

Class: XII Time: 3 HOURS

Subject: Mathematics Max. Marks: 100

SECTION – A (6 x 1 = 6)

01. If f: R→R is invertible defined as f(x) = 3x+2, find (f ∘ f)(x).

02. Evaluate: $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$

03. What is the principal value of $\tan^{-1}(-1)$?

04. Write A^{-1} for A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

05. If a matrix has 6 elements, write all possible orders it can have?

06. Evaluate: $\int \frac{(1+\log x)^2}{x}dx$

SECTION – B(13 x 4 = 52)

7. Find the image of (1,2,3) about 2x – 3y + 4z = 7

8. How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80%?

(OR)

A biased die is twice as likely to show an even number as an odd number. The die is rolled 3 times. If the occurrence of an even number is considered as a success, then mean number of success.

9. Show that the four points (0, -1, -1), (4, 5, 1), (3, 9, 4) and (-4, 4, 4) are coplanar.

Also find the equation of the plane containing them.

10. Solve the differential equation: $(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}$ given that y = 0 when x=1

11. Consider $f:R_+\to[4\ \infty]$ given by $f(x) = x^2 + 4$. Find f^{-1}if f is invertible.

12. Prove that: $\frac{9®}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

(OR)

Solve: $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}(x)$; x > 0

13. Using the properties of determinants, Prove that

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

14. If y = $\log\left[x + \sqrt{1+x^2}\right]$, prove that $(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

(OR)

If $x = a(¯ - \sin ¯)$ and $y = a(1 + \cos ¯)$ then find $\frac{d^2y}{dx^2}$.

15. Find the value of ‘a’ if the following function is continuous at x = 0

$$f(x) = \begin{cases} a \sin \frac{®}{2}(x+1), & x \le 0 \\ \frac{\tan x - \sin x}{x^3}, & x \succ 0 \end{cases}$$

16. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangent are parallel to X-axis.

17. Using differential, find the approximate value of $\sqrt[4]{258}$

18. Evaluate: $\int \frac{2x\,dx}{(x^2+1)(x^2+3)}$

(OR)

Evaluate: $\int e^{2x} \sin x\,dx$

19. Solve the differential equation: $ye^{\frac{x}{y}}dx = (xe^{\frac{x}{y}} + y)dy$

<u>SECTION – C(7 x 6 = 42)</u>

20. Using matrix method, solve the following system of equations:- x - y + 2z = 1; 2y - 3z = 1; 3x - 2y +4z = 2 (OR) Use elementary transformations, find the inverse of the matrix $\begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$

21. Find the equation of the plane passing through the point (1, 1, 1) and containing the line $\vec{r} = (-3\vec{i} + \vec{j} + 5\vec{k}) + \text{ª}(3\vec{i} - \vec{j} + 5\vec{k})$. Also show that the plane contains the line $\vec{r} = (-\vec{i} + 2\vec{j} + 5\vec{k}) + \text{ª}(\vec{i} - 2\vec{j} - 5\vec{k})$

(OR)

Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.(\vec{i} + 2\vec{j} + 3\vec{k}) - 4 = 0$ and $\vec{r}.(2\vec{i} + \vec{j} - \vec{k}) + 5 = 0$, which is perpendicular to the plane $\vec{r}.(5\vec{i} + 3\vec{j} - 6\vec{k}) + 8 = 0$

22. A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10 metres. Find the dimensions of the rectangle so as to admit maximum light through the whole opening.

23. Evaluate: $\int_0^{\frac{\text{®}}{2}} 2\sin x \cos x \tan^{-1}(\sin x)dx$

24. Using integration find the area of the triangular region whose sides have equations

y = 2x + 1 ; y = 3x + 1 ; x = 4

25. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of grinding/cutting machine and a sprayer. It takes 2 hrs on the grinding/cutting machine and 3 hrs on the sprayer to manufacture a pedestal lamp. It takes 1 hr on the grinding/cutting machine and 2 hrs on the sprayer to manufacture a wooden shade. On any day the sprayer is available for at the most 20 hrs and grinding /cutting machine for at the most 12 hrs. The profit from the sale of

a lamp is Rs. 5 and that from a shade is Rs. 3. How many of each should be manufactured so as to make maximum profit? Make an L.P.P and solve graphically.

26. A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further 2% of items produced by machine A and 1% of items produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?

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KENDRIYA VIDYALAYA NO 1 TAMBARAM

PRACTICE EXAMINATION

Class: XII Time: 3 HOURS
Subject: Mathematics Max. Marks: 100

SECTION – A (6 x 1 = 6)

01. If f: R→R is invertible defined as $f(x) = x^3 + 3$, find $f^{-1}(x)$.

02. From the following matrix equation, find the value of x

$$\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

03. What is the domain of $\cos^{-1}x$?

04. Evaluate the determinant $\begin{vmatrix} 4 & a & b+c \\ 4 & b & a+c \\ 4 & c & a+b \end{vmatrix}$

05. If A is a non-singular matrix of order 3 and |adj A| = $|A|^k$, then find the value of k

06. Evaluate: $\int \frac{\sec^2 x}{3+\tan x}dx$

SECTION – B (13 x 4 = 52)

07. Find distance of (1,2,3) from 2x – 3y +z = 7 measured perpendicular to

3x + y – 2 z = 8

08. Solve the differential equation: $x\log x\frac{dy}{dx} + y = 2\log x$

09. Find the shortest distance between the lines $\vec{r} = (4\vec{i} - \vec{j}) +$ ª $(\vec{i} + 2\vec{j} - 3\vec{k})$ and

$\vec{r} = (\vec{i} - \vec{j} + 2\vec{k}) +$ « $(2\vec{i} + 4\vec{j} - \vec{k})$

10. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of no of Aces and find the mean and variance.

11. Let ‘*’ be a binary operation on N defined by a * b = HCF (a, b). Is * is commutative and associative? Does there exist identity element for the binary operation ‘*’ on N?

12. Prove that: $\tan\left[\frac{®}{4}+\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right]+\tan\left[\frac{®}{4}-\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right]=\frac{2b}{a}$

(OR)

Prove that: $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)=\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

13. Using the properties of determinants, Prove that

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$$

14. If y = $\sqrt{\frac{\sec x-1}{\sec x+1}}$, find $\frac{dy}{dx}$ at x = $\frac{®}{2}$

(OR)

If y = $\sqrt{x^2+1}-\log\left(\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}\right)$, find $\frac{dy}{dx}$

15. Discuss the continuity of the function f(x) given by

$$F(x) = \begin{cases} \frac{1-\cos x}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

16. Find $\frac{dy}{dx}$, if $y^x + x^y = k$, where k is constant.

17. Using differential, find the approximate value of $\sqrt[3]{29}$

(OR)

Find the intervals in which the function f(x) = $(x-1)(x-2)^2$ is (i) increasing (ii) decreasing.

18. Evaluate: $\int_1^2 (3x^2-1)\,dx$ using limit of sums

(OR)

Evaluate: $\int_0^{\frac{®}{2}} \log\sin x\,dx$

19. Solve the differential equation: $(x^2-y^2)\,dx + 2xy\,dy = 0$, given that y = 1 ,when x = 1.

SECTION – C (7 x 6 = 42)

20. Evaluate $\int \frac{x^2 - 3x}{(x-1)(x-2)} dx$

21. Find the area of the region bounded by the curve $x^2 = 4y$ and $y^2=4x$

22. There are two bags, Bag 1 and Bag 2. Bag 1 contains 4 white and 3 red balls white another Bag 2 contains 3 white and 7 red balls. One ball is drawn at random from one of the bags and it is found to be white. Find the probability that it was drawn from Bag1.

23. Using matrix method, solve the following system of equations:-

$3x - 2y + 3z = -1$; $2x + y - z = 6$; $4x - 3y + 2z = 5$

24. Find the equation of the plane passing through the point (-1, 3, 2) and perpendicular to each of the planes $x + 2y + 3z= 5$ and $3x + 3y + z = 0$.

(OR)

Find the coordinates of the foot of the perpendicular and perpendicular distance of the point P(3, 2, 1) from the plane $2x – y + z + 1 = 0$

25. Show that the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

(OR)

Show that the height of the cylinder of maximum volume that can be inscribed in a Sphere of radius 'R' is $\frac{2R}{\sqrt{3}}$.

26. One kind of cake requires 200g of flour and 25 g of fat and another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat, assuming that there is no shortage of the other ingredients used in making the cakes. Formulate the above as L.P.P and solve graphically.

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